DIRICHLET SERIES.

The General Theory of Dirichlet Series. By G. H. Hardy and Marcel Riesz. Pp. x+78. (Cambridge: At the University Press, 1915.) Price 3*s.* 6*d.* net.

IT is well known that Dirichlet made a new start in the theory of numbers by bringing it into connection with certain analytical functions of the form $\sum_h n_h^{-s}$, n_h being an integer, and s a real quantity. In the present tract the definition of a Dirichlet series is

$$f(s)=\sum_1^\infty a_n \exp(-\lambda_n s),$$

where (λ_n) is a sequence of real increasing numbers converging to $+\infty$, and s is a complex variable. This obviously includes Dirichlet's functions, and also those considered by Dedekind, Riemann, Landau, and others, in the same connection.

In the earlier chapters we have a summary of the known elementary properties of $f(s)$, such as the conditions of its convergence when regarded as a sum in the ordinary sense, and so on. But the most novel, and principal, part, due mainly to Dr. Riesz, is concerned with what he calls the summability (λ, κ) of $f(s)$. It is important that the meaning of this should be properly understood. Let s_n mean the sum of the first n terms of $f(s)$; then Dr. Riesz associates with this a function which we may call $\sigma_n(\lambda, \kappa)$, where κ is an arbitrary positive number, and λ is a functional symbol defined by $f(s)$ and κ, so that we have a sequence $[\sigma_n(\lambda, \kappa)]$ which we may consider as depending upon κ. If this new sequence has a limit L, we say that $f(s)$ is summable (λ, κ). In the definition we only consider the *formal* nature of $f(s)$, so that it does not matter whether $f(s)$, regarded as a series in the ordinary way, is convergent or not. Thus we have a sort of extension of the theories of Poincaré, Borel, etc., about divergent series.

The conditions that L may exist give properties of $f(s)$ considered as a formal expression, and hence certain conclusions of an arithmetical nature can be drawn. It is in the further development of these deductions that we may hope for further information of interest. The mere fact that from a sequence (s_n) we may be able to construct a sequence (σ_n) which agrees, in the limit, with (s_n) when the latter is convergent, and may be called its "sum" when it becomes divergent in the ordinary sense, is a barren definition until we apply it to something concrete; if we can do this, it may be a very valuable resource, as in the case of asymptotic summation. Everything goes to show that the + we use in writing infinite series is in some ways less appropriate than the comma of the old-fashioned "progressions"; it will not matter if we remember what we are doing, and that there is no such thing as the actual sum of an infinite number of terms.

One observation in this tract is liable to misunderstanding. It may be true that Cahen is the first to discuss $f(s)$ *systematically* as a function of a complex variable s; but the first step in this direction was taken by Riemann in his famous paper on the distribution of primes. To arithmeticians, at any rate, it is the development of Riemann's results that is of principal interest at present; other problems of the same kind naturally present themselves, as, for example, the frequency of cases where p and $(p+2)$ are both primes, like (5, 7) or (11, 13). The new analysis may throw some light on these and other dark places. In any case, this tract will be welcome for its concise statement of known facts, and its bibliography, which supplements that of Landau.

G. B. M.

OUR BOOKSHELF.

Fungoid Diseases of Farm and Garden Crops. By Dr. T. Milburn. Pp. xi+118. (London: Longmans, Green and Co., Ltd., 1915.) Price 2*s.* net.

THIS book is intended "primarily for the use of farmers, gardeners, and agricultural students," and it is hoped that it may assist also "those engaged in teaching and county lecturing." The essentials of a book fulfilling these aims must be that for the student the elementary scientific facts are correctly and clearly stated, and for the farmer and gardener that his understanding of the subject is developed by his attention being directed to general principles while practical help is given. This book falls short of this standard.

The farmer and gardener fail to get the knowledge they want when they are told that "no exact formula" can be given for making Bordeaux mixture, and that "it is always necessary to test the solution before applying, for if too much lime be present it is useless as a fungicide." While the author quotes the titles of recent works on the chemistry of Bordeaux mixture, it seems scarcely possible that he has read them from the account he gives of the chemical composition of Bordeaux mixture. The statement that the preparation of soda-Bordeaux is "somewhat critical," and that since this mixture "possesses no effectual advantage" it can be passed over, will considerably surprise the Irish Department of Agriculture and the numberless farmers who, under their tuition, have saved thousands of acres of potatoes from "blight" by the use of this "soda-Bordeaux" or "Burgundy mixture." The gardener and fruit-grower will not acquiesce in the strange statement that "washes of potassium sulphide" are "very effectual, but too expensive."

The book is well printed and very cheap; it would have been better, however, to have in-

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creased its cost by greater care being given to its compilation and by enriching it with good reproductions of photographs of diseased crops.

A First Book of School Gardening. (First Books of Science.) By A. Logan. Pp. vi+151. (London: Macmillan and Co., Ltd., 1915.) Price 1*s.* 6*d.*

THE aim of school gardening in this country is too often merely to teach children to grow cabbages (and other crops) successfully. The author of this little volume, who is well known as a pioneer of school gardening in the north of Scotland, would, in addition, "instruct pupils in the fundamental principles of soil management and plant growth." This double aim is carried out by means of classroom lessons and practical work. "Neither of these types of work, however, has been allowed to encroach upon the other." The book is much more handy in size than the author's previous volume, and is better for being more concise. It will be welcomed by numerous teachers as being exactly what they want.

LETTERS TO THE EDITOR.

[*The Editor does not hold himself responsible for opinions expressed by his correspondents. Neither can he undertake to return, or to correspond with the writers of, rejected manuscripts intended for this or any other part of* NATURE. *No notice is taken of anonymous communications.*]

Modern Universities in China.

EVERYONE knows that the Chinese once led the world in scientific and material development. What may surprise Europeans who are unacquainted with the Far East is that they understood the principles of good engineering design long before our learned men in Britain had investigated such matters.

The arch, beautiful from the scientific as well as the æsthetic point of view, is frequently to be seen in parts of China which have only recently been explored by the white man. The undoubted antiquity of many existing examples proves that this type of structure antedates the arrival of the foreigner. Like printing and gunpowder and many other inventions and discoveries, it was probably of Chinese origin. The bridges, the pagodas, the city walls, and certain details of building construction to be seen in China demonstrate that many centuries ago there were men in that wonderful country who had a complete understanding of some most important principles used in applied science work.

Unfortunately there came a time when classical education alone was in favour. The disasters, political and economic, which have fallen upon the Chinese during their recent history have beer in no small measure due to the paralysing effect of their educational system. About thirty years ago a very few people in authority in Peking realised this fact. During the last ten or fifteen years there have been violent changes in the outlook of millions of the people. The old ideas concerning education are almost dead. Western knowledge is in demand.

Many Chinese have entered the universities of Europe, America, and Japan. An effort has also been made to create universities in China. Anyone who has taken an interest in the lives of Oriental students in England must come to the conclusion that it is better to carry Western learning to China, rather than send Eastern students to Europe.

The number of modern universities in China is not great. There are some under the control of the various missionary societies, but of these there is only one (St. John's University, Shanghai) which has any real claim to the title. Other institutions, with praiseworthy ambitions but poor finances, connected with the work of evangelisation are the North China University, the Soochow University, and those known as the Shantung Christian, the Nanking, and the Boone (Wuchang) Universities.

The three which are under entirely secular control are the University of Hong Kong, the Government University of Peking, and the Peiyang University, Tientsin.

An important visitor to any of these centres of learning would come to the conclusion that there is only one of them which is entitled to be ranked with the provincial universities of England. The missionaries have not sufficient funds to obtain the necessary staff and equipment. The Chinese Government place the administration in the hands of Chinese officials, which is almost hopeless. Corruption still exists in high places, despite the changes due to the revolution of 1911.

The University of Hong Kong has already established itself as worthy of British traditions. It commenced its career in 1912, and after three years it has earned the confidence of both Chinese and Europeans in the Far East.

Two events which have recently occurred are significant. President Yuan has provided two thousand dollars per annum for scholarships, tenable by Chinese subjects. A wealthy Chinese, resident for many years in the Straits Settlements, has recently given the University a sum of money equivalent to about 30,000*l.* The conditions of his bequest are curious. He has lent the University half a million dollars, without interest, for twenty-one years.

A few weeks before this offer was received, a citizen of Hong Kong, named Ho Tung, presented the University with one hundred thousand dollars.

In order to show that the local Chinese are sympathetic to, and interested in, this centre of learning, it may be mentioned that a few months ago the writer obtained from them the promise of about one thousand pounds for a cricket pavilion for the Students' Union. This money was raised in a few days, despite the fact that everyone in the colony has given very freely to the various war funds.

The University buildings were presented by a wealthy Parsee, the late Sir Hormusjee Mody. The Government gave the site. An endowment fund, exceeding one million dollars, was collected by the Chinese. Messrs. John Swire and Sons, and affiliated firms, provided 40,000*l.*, and the first chair to be endowed in the University is called the Taikoo chair of engineering, as the Chinese name for this important mercantile house is Taikoo. Various other British firms in the Far East subscribed to the original endowment fund. The Government of the colony contributes 2000*l.* per annum, which the Chinese members of the Legislative Council think should be increased.

After an experience of three years, the University authorities have every reason to be satisfied with their work. There are now about two hundred undergraduates. The tuition fees are the equivalent of thirty pounds per annum, and, in addition, the student must pay for his board and lodging in one of the five hostels or colleges. It probably costs a parent about 100*l.* per annum to send his son to the University. This compares favourably with the cost of sending him to Europe or America. The engineering and arts

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